MODELLING IN AN INTEGRATED MATHEMATICS AND SCIENCE CURRICULUM: BRIDGING THE DIVIDE

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This paper explores the theoretical rationale behind a new approach to developing integrated mathematics and science curriculum experiences using the construct of bridging concepts. Such interdisciplinary approaches provide challenges for teachers and learners as they need to develop new practices in their classrooms and communicate across traditional subject boundaries whilst ensuring curriculum objectives in both subjects are met. Experiences from students across case study schools suggests that they need considerable time and space in which to develop conceptual understanding within and across mathematics and science and the vocabulary with which to communicate their thinking. The development to-date points to an important role for applications and modelling in mathematics in such cross-curriculum learning.

BACKGROUND:

The place of mathematical modelling in curricula of compulsory schooling is often contested and even with the pressure of international assessment such as PISA giving priority to applications of mathematics it may struggle to find a place (Ofsted, 2008). Equally scientific enquiry, as part of what might be considered a ‘reform’ agenda in science, that prioritises scientific method and consequently sees the need for enquiry as central, prioritises approaches that are not always prominent in day-to-day teaching (for example, Sadeh and Zion (2009), Wilson et al (2010)). The work described here relates to an EU funded Comenious project COMPASS (Common problem solving strategies as links between Mathematics and Science) which seeks to tackle these issues promoting an integrated approach to the learning of mathematics and science by developing a range of classroom materials and professional development support for teachers. Fundamental to this project is the development of approaches to teaching that motivate learning both within and across the disciplines. Pragmatically, given that there is no official demand for this (at least in England), this requires some understanding of how such innovation can be motivated in ways that ensure that current curriculum demands are being met.

Here, therefore, I present a theoretical rationale behind a new approach to interdisciplinary teaching and learning in mathematics. The outcomes of an initial adoption of materials based on this are reflected upon so as to inform a next iteration of their further development. I firstly explore how such aims might be achieved by considering, albeit briefly, the nature of the different domains of mathematics and science and exploring different ways that the demands of each, and indeed the demands of teachers and learners of each, might be met. I present a theoretical overview and a rationale of the design of interdisciplinary teaching and learning that, therefore, is also cognisant of these analyses of the subject domains of mathematics and science informed by an understanding of interdisciplinary learning approaches. Additionally the resulting approach also has to meet the aims of the COMPASS project that seeks to see mathematical and scientific activity situated in contexts that are meaningful to critically informed European citizens of the future. The approach adopted is exemplified in this paper by outlining a set of materials that have gone through a first cycle of design and improvement in collaboration with mathematics and teachers working in a small group of schools in England. Finally, I draw on case study data and particularly focus group interviews with teachers to reflect further upon the design principles and implications for the integration of mathematics and science and the potential importance of applications and modelling in such curriculum design.

DEVELOPING A THEORY INFORMED DESIGN APPROACH

Nikitina (2006) identifies, on the basis of empirical classification, three basic approaches to the development of interdisciplinary approaches: contextualising, conceptualising and problem – centering. Her classification suggests that in general underlying differences and similarities in pedagogies and epistemologies in different disciplines are likely to result in one or other of these different approaches being adopted when school subjects are considered in common patterns of interdisciplinarity. For example, in the case of interdisciplinary approaches involving (1) disciplines in the humanities the common approach is often one of *contextualising*, that is one of developing tasks and materials that pay attention to issues of time, culture and personal experience. This contrasts with likely approaches to interdisciplinary work (2) in the scientific disciplines that is likely to focus on *concepts* that are central to constituent disciplines and work in quantifiable ways in making connections and (3) takes a critical *problem-solving* approach to issues of social concern that may require a more eclectic combination of knowledge and skills across disciplines. This would suggest that an interdisciplinary approach to study of mathematics and science is commonly one that focuses on concepts. It is perhaps not surprising that Nikitina’s classification is reflected in other more philosophical analyses of mathematics and science knowledge domains that attempt to capture the essence of these in school subjects.

For example, Figure 1, adapted from the PISA overview of the mathematical literacy domain (OECD, 2003), draws attention to important components to take into account when considering this: concepts, competencies and contexts or situations in which these might be situated. These in turn inform formulation of tasks and ultimately learner experience. The looping connections in the diagram suggest that there is no straight forward heuristic that can be adopted in developing tasks and approaches to lessons, but rather that the process is inevitably fuzzy as the different factors are brought to bear on classroom experiences.

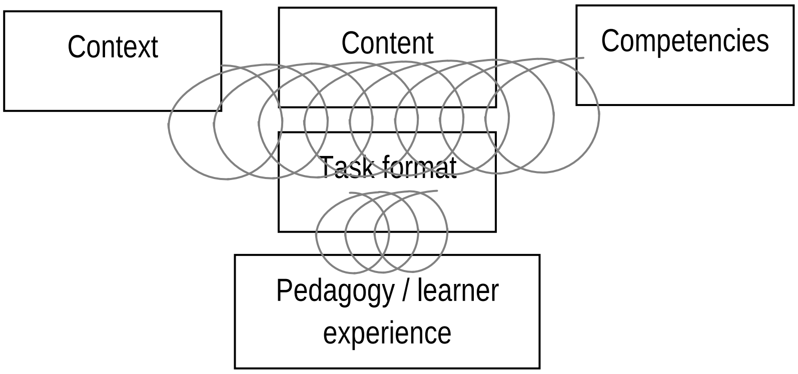


Figure 1. School mathematics domain (Wake, 2010) based on PISA analysis (OECD, 2003)

The influence of the OECD international comparative study (PISA) that has over recent years measured student performance at age 15 in mathematics, science and literacy has been important in informing curricula design across nations (Grek, 2009). This is the case in England where in 2007 (QCA, 2007) a new national curriculum for all subjects was introduced. In mathematics this gave greater prominence to competencies / process skills than had hitherto been the case refelecting the greater emphasis given to these by the PISA tests. The key processes of *representing*, *analysing* (using procedures and reasoning), *interpreting* and *evaluating* have been emphasised in relation to a problem solving cycle (see Figure 2) which is based on the modelling schema included in the OECD framework (OECD, 2003).

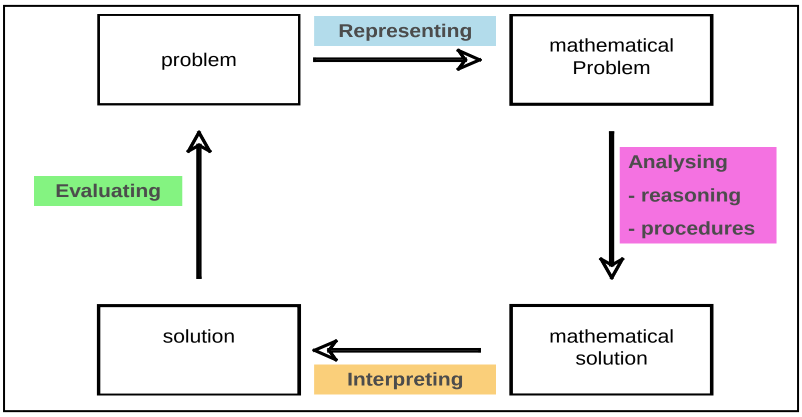


Figure 2. Problem solving / modelling cycle in the English mathematics curriculum

The problem solving cycle as represented in the curriculum is more general than in PISA as it is expected that students could be working in the world of mathematics itself rather than working towards solution of a problem in reality (as in PISA). The corresponding framework for the scientific literacy domain equally identifies the importance of the three components; (knowledge /) *concepts*, (processes /)

competencies and (situations /) *contexts*. As in the case of mathematics, science in the school curriculum tends to focus on content and competencies at the expense of context with the emphasis on the former of these components. In science discussion of concepts identifies thirteen major and diverse themes ranging from the property of matter to genetic control. Scientific processes are grouped under three main themes: (1) describing, explaining and predicting scientific phenomena, (2) understanding scientific investigation, (3) interpreting scientific evidence and conclusions (OECD, 2003, p137).

In summary, school mathematics and science practices may, therefore, be considered as being culturally and historically situated primarily in the *concept* component of the domain with recent curriculum formulations in England having the intention of shifting the focus to the *competency* component. This shift in focus of curriculum priority suggests a potential way forward in interdisciplinary curriculum design would be to focus on common mathematical and scientific competencies

However, there remains one component of the subject domain that as yet has not been discussed in relation to its potential to support interdisciplinarity: that of *contexts*. This is often neglected in both mathematics and science which as school disciplines may be seen as rather abstract areas of study with only hints of their applications and usefulness in contemporary contexts. There have been recent attempts in science to remedy this with courses such as 21st Century Science (Millar and Osborne, 1998) attempting to situate scientific understanding more realistically in the lives of the students who are expected to study this. It should be noted that such courses are not without their critics who see them as diluting true / ‘academic’ scientific knowledge and understanding. The role of *contexts* in the interdisciplinary approaches to be developed is considered important by the COMPASS project that seeks to provide teachers with rich, motivating materials that allow young people to explore meaningful problems in a European context. In other words an important aim of the project is to motivate students by posing problems that can immediately be seen to have importance to young people as citizens of Europe with a concern for their environment and the population that inhabits this.

Overall, then, in designing materials for classroom use in ways that bring together meaningful learning in both mathematics and science as separate disciplines as well as in ways that makes powerful interconnections it was the COMPASS group’s desire to pay due attention to each of the components (contexts, content and competencies) of the discipline domain without privileging any one at the expense of the other. In an attempt to do this whilst also addressing the pragmatic issue of providing neutral ground on which mathematics and science teachers might jointly work together we developed the construct of the “bridging concept”. This is an organising structure that draws on mathematical and scientific thinking in ways that provide a way of visualising and understanding how measurable and therefore quantifiable phenomena (interact or) behave. Importantly, Nikitina’s analysis suggests that this is likely to meet with least resistance from mathematics and science teachers who appear to be most comfortable with interdisciplinary approaches of a type that gives prominence to concepts. However, before going on to describe bridging concepts more generally and exemplify in some detail one such concept I wish to emphasise that the materials that we eventually produced pay due attention to the other important components of the discipline domains. The bridging concept, therefore, attempts to take a new epistemological approach to knowledge across science and mathematics by looking for ways in which understanding phenomena might be considered as being general and consequently having applications across many situations / contexts. For example, the need to understand how outcomes at the micro level are scaled up and have implications at the macro level appears important in many situations. Consider, for example, issues (1) of inoculation against disease: how does an individual's decision to be inoculated or not impact on the likely spread of disease at the level of the population? (2) of the impact of a switch to energy saving devices / equipment such as light bulbs: what are the implications for the individual and for society as a whole? To investigate these problems deriving from context driven issues clearly requires the use of both concepts and competencies from mathematics and science but it is the intention that at a meta-level students also have opportunities to consider how micro-level actions have implications and outcomes at a macro-level. It is the task formulation (Figure 1) that determines how concepts and competencies in mathematics and science will interact. At this stage we have not identified a substantive number of such bridging concepts but the examples and teacher and student reactions that I go on to discuss focus on a further bridging concept: that of *flow*. Again there are multiple needs to understand flow across the different sub-disciplines of science and indeed more widely in technology-based settings: flow is a central characteristic of contexts ranging from electric circuits to ecosystems, from the human heart to tidal estuaries, from solar emissions to traffic management. When linked with flow, the concept of equilibrium is a key to understanding stability in systems as varied as electricity supply, transport, geothermal activity and ecosystems.

In summary, therefore, bridging concepts seek to value concepts from both the science and mathematics domain in making sense of phenomena and have general applicability at a meta-level. Importantly they provide a driver to facilitate cross-disciplinary thinking in ways that are deeper / richer than would otherwise be the case. The intention is that the bridging concept gives new insights to teachers and learners alike at how they can use mathematics and science to make sense of a range of issues of importance to European citizens of today and the implications of these for the future.

BRIDGING CONCEPT: FLOW

The materials developed and exemplified here focus on the bridging concept of flow and supported students towards a contextual problem based on flooding. "How can further disastrous flooding and environmental damage be prevented? – a case study of Lynmouth in Devon, UK." However, this task comes as the culmination of a substantial programme of lessons in both mathematics and science that focused on key underpinning conceptual understanding. The flow of liquid was central throughout this particular set of materials although at the beginning the need to understand flow as an important concept was motivated by video clips that allowed students to consider flow in the context of traffic and crowd movement.

Here due to restrictions of space I will exemplify the teaching materials with reference to mathematics materials only, before returning to consider implementation issues that arise from case studies of classrooms in a number of schools.

The mathematics is organised around two web-based *research environments* or *applets,* which students are encouraged to use to explore in a systematic way how different parameters affect aspects of measuring / quantifying and controlling flow. These applets model, in an idealised way, situations that the students will need to understand when they come to consider flow in science (for example, how flow is affected by the permeability of materials) and eventually the flow of rainwater that may give rise to flooding. In working with the applets students are expected to *predict – test – explain***,** that is, to pose a question and predict the outcome before they use the applet to test this, and then explain why their prediction was correct or not, following the problem solving cycle with questions posed and predictions of outcomes important in concept development.

The first applet "Roof" simulates rain falling on a roof and draining into a blocked gutter. This provides an opportunity for students to come to an understanding that water, being a liquid, can flow from one place to another, and that we can measure / calculate its volume when it collects in a container. This provides an idealised scaled down model of a hillside with rain draining into a river that students will need to consider at a later stage. The applet model assumes that all of the water on the roof drains into a gutter on each side of the roof (Figure 3), and that the gutters are blocked so that all of the water collects there and is distributed evenly throughout the gutter. The gutter is assumed to be horizontal and that the depth of water in all parts of the gutter is the same throughout. The *applet* allows students to vary the total amount of rainfall, the angle of slope of the roof and the width of the house with graphs showing how each of these affect the height of water in the gutter. Using a trace facility allows the user to see how each variable is affected as they alter, using the slider, any one of the parameters between its minimum and maximum values. Although it is possible for students to carry out calculations of area, volume and capacity it is also possible for them to pose questions such as, "What would happen if the width of the house was doubled, multiplied by three, halved and so on?"

A second applet simulates water flowing into and out of different cuboid and trapezoidal containers. In these simulations it is assumed that flow rates are constant throughout and that the containers are horizontal so that depths of water are constant throughout the container at all times. Having selected either a rectangular or trapezoidal container of volume 10, 20 or 30 units, and selected flow rates for water entering and leaving the container, graphs, on one set of axes, can selected to be plotted of (i) total water flow in, (ii) total water flow out, (iii) volume of water in the container and (iv) total volume of water overflow from the container against time. A separate additional graph of height of water in the container against time is plotted. Again students are expected to explore in a systematic way, and again “predicting – testing and explaining”, how the flow through a ‘container’ is affected by important parameters. This simulation can be used to model water flow through guttering from a roof or from a hilly landscape through a river or drainage channel.

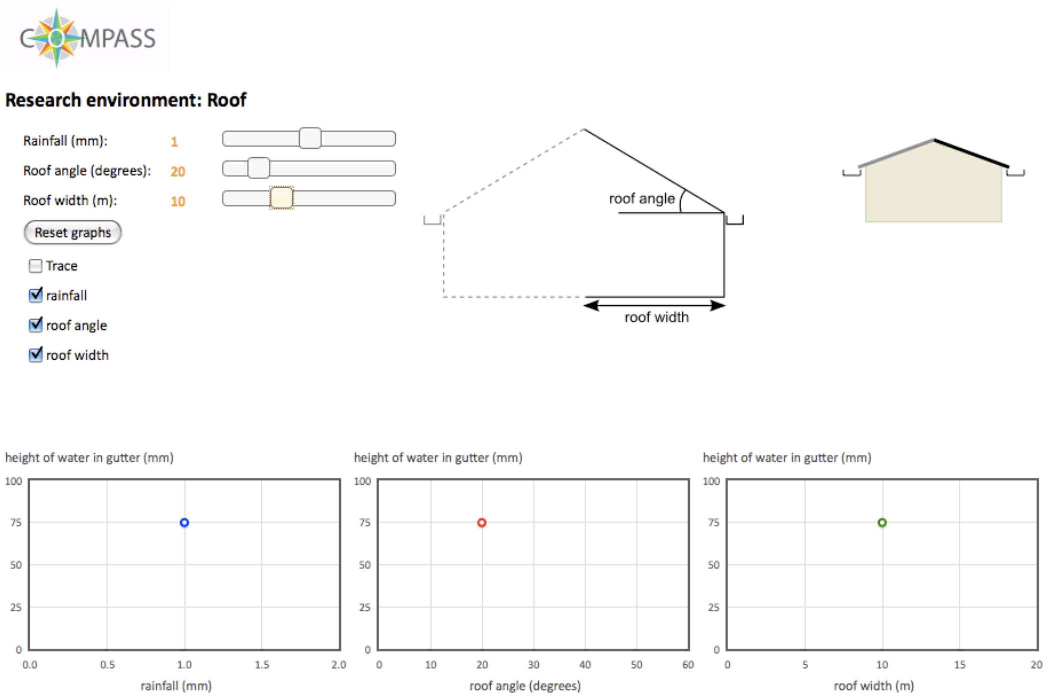
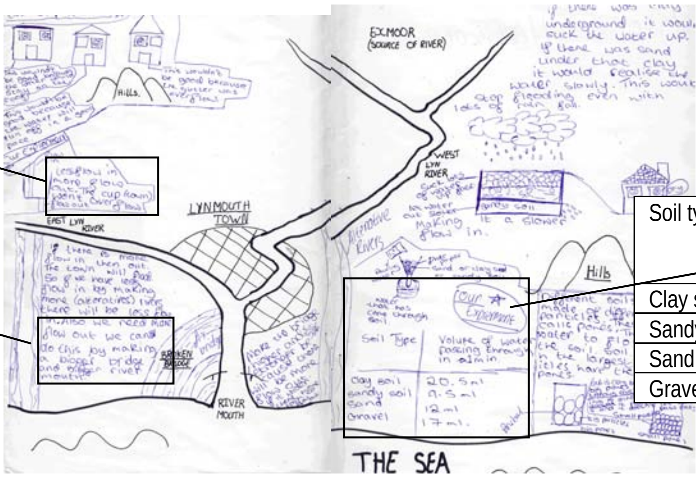


Figure 3. Applet used to investigate the collection of rainwater from a roof

As a final part of the sequence of activities students were asked to engage with a substantial problem that asked them to think about how flooding might be prevented in a valley leading to the sea: this scenario was based on an event in Lynmouth in Devon in 1952 where flash flooding caused substantial damage. Students were asked to use the results of some of the experiments they had carried out in maths and science to give the best possible advice to (a) farmers and others using and draining the land on the moors above Lynmouth; (b) bridge engineers planning to rebuild bridges in the valley; (c) engineers planning how the river should flow through the town; (d) people wanting to rebuild their houses and homes in Lynmouth. It was noticeable that much of the output of the students was descriptive rather than involving calculations. Some of this is shown in Figure 4.



Soil type	Volume of water passing through in 1 minute
Clay soil	20.5 ml
Sandy soil	9.5 ml
Sand	12 ml
Gravel	17 ml

Figure 4. Student accounts in response to flooding problem

In England the approach and materials explained in this paper have been used in a first cycle of development by a mathematics and science teacher from each of eight different schools. The teachers' work and accounts of their experiences with their students in using the materials has been facilitated by two days of professional development workshops generously funded by an IMPACT grant from the Science Learning Centre North West: additionally over a period of two weeks each school had the support of a newly qualified mathematics and science teacher. In general each of the schools worked with pupils in both mathematics and science using the materials. The following reflections on the outcomes of this implementation are informed by focus groups that were convened as part of the second of the workshops during which teachers were asked to reflect on nine key areas that explored the design approach and their implementation of this.

REFLECTIONS AND THE ROLE OF MODELLING

In considering the students' work in mathematics many of the teachers drew attention to ideas of modelling although the applets effectively present them and their students with ready-made models which they were able to use to explore issues that would eventually relate to a more complex reality. This type of exploratory environment therefore might be classified as being of the perspective "Educational modelling type (b) conceptual modelling" in the classification system as proposed by Kaiser et al (2007). The applets allow quantifiable aspects of the situation to be varied and the effects of doing so to be explored systematically: however, other potential variables are fixed to ensure a relative simplicity of the model and consequently provide for a didactical focussing on key concepts and competencies. For example, in the use of both applets students were encouraged to "predict-test-explain" with teachers reporting that this enquiry based activity resulted in a much more dialogic classroom with students taking the lead in directing activity and working in collaborative groups. The conceptual understanding of issues relating to flow encapsulated in the applets appears to have been of fundamental importance to students learning to use these to inform their work in science and in tackling the substantial modelling task at the end of the sequence of lessons. Perhaps it is not surprising that students did not focus primarily on quantitative work when a more qualitative conceptual understanding had been sought by asking them to predict outcomes and graphical representations of these when working with the applets.

A crucial question for the design of the interdisciplinary approach I report here is the role that the bridging concept (in this case “flow”) played. In focus group interviews teachers reported that this was important (i) for themselves as teachers because it provided them with a new way of thinking / working together as a team developing a common frame of reference for participants, (ii) for students because it provided a new starting point and setting in which to work with familiar ideas to see things differently, (iii) in mathematics as it involved students in modelling and applications, and (iv) in science because it provided a common overview by which links and connections across the curriculum could be made. It appears, therefore, that the bridging concept provides a boundary object (see for example, Tuomi-Grohn and Engestrom, 2003) at the intersection of the usual formulations of the mathematics and science curricula. It seems that this allows students and teachers means by which they might develop a new epistemology that brings together knowledge and competencies in mathematics and science in ways that interplay with and reinforce each other. Our work with teachers and students in schools to date has highlighted that this requires students time and space in which they can explore key ideas (in this case related to flow) that allows them to develop conceptual understanding and gives them language and terminology by which they can start to communicate with each other about relatively complex ideas. The applets we developed in the case reported here (and which we have developed for other units of work) appear to provide a vehicle by which students might be able to develop this necessary understanding. However, we note that the understandings developed are indeed at a more conceptual level than being quantifiable / calculable (for example, students have considered what might occur if both flow-in and flow-out were doubled without worrying about specific rates of flow). We also note that the modelling work produced by students drew substantially on this conceptual understanding rather than leading naturally to an approach that resulted in careful calculation.

We are lead to conclude, at this stage, that in developing an interdisciplinary approach to mathematics and science the construct of ‘bridging concept’ is useful as it allows both teachers and students new approaches to developing knowledge that synthesises mathematics and science. In such approaches to learning we have found that students need support in understanding new concepts and language with which to discuss these. We have found particularly useful purpose built applets that students can use to explore how varying different parameters can affect a mathematical model of a situation. These applets involve students in developing a range of modelling sub-competencies as well as engaging with important mathematical content, providing a playful environment in which a pre-constructed model can be used with purpose to explore potential outcomes relatively quickly. It is to be hoped that these new concept focused approaches in mathematics may ultimately have spin-off as and when students come to build their own, and quantifiable models of situations.

In general, therefore, early indications are that bridging concepts have the potential to inform the design of materials that may support an interdisciplinary approach to the learning of mathematics and science content and competencies in meaningful contexts. For mathematics in particular they give a motivating purpose for more conceptual, dialogic classrooms with mathematical models central.

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